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casting a rod upon a ruled surface as above described. And as he claimed that he had avoided all conceivable sources of bias in pitching the rod, it was expected that the mean of so great a number of experiments would agree very nearly with the theoretical value of P; but, contrary to expectation, though the mean of the first 3000 experiments gave almost exactly the theoretical value of P, from that point onward to the end of the 30,000 trials the experimental result steadily diverged from the theoretical value of P.

In the discussion which followed the reading of this paper, various reasons were assigned for the discrepancy between the theoretcal and practical results, none of which seemed satisfactory to the Section. Among other possible sources of error in the result of the trials a personal eq'n was suggested by Prof. Mendenhall. The source of the error in this case, however, will, most likely, be found in that physiological characteristic of muscular action in accordance with which any act, frequently repeated, is continued unconciously in the same way. As this peculiarity cannot be eliminated by a personal equation, it follows that, in experiments of this kind, instead of approaching the theoretical value indefinitely by extending the experiments to a very great number, the liability of falling into special habits after a cert'n number of experiments have been performed not only renders further experimentation, by the same individual. useless, but even prejudicial.

INTEGRATION OF SOME GENERAL CLASSES OF TRIGONOMETRIC FUNCTIONS.

BY PROF. P. H. PHILBRICK, IOWA STATE UNIVERSITY, IOWA CITY.

In solving some of the more difficult problems that naturally arise in the higher branches of Applied Mathematics and in connection with a general course in engineering, one frequently has occasion to perform integrations, the formulas for which are not found in the books, at least not in those accessible to the operator.

Some recent experience of this nature led me to integrate some very general classes of Trigonometric functions to which those I incidentally encountered belong, and I offer the results and the methods of arriving at them for such consideration as they deserve. It is perhaps needless to remark that the subsidiary formulas required, are developed and used without special enquiry in regard to their existence elsewhere.

Integration of $\dfrac{dx}{(a+b\sin x)^n}$.

$$\int\frac{dx}{(a+b\sin x)^n}=\int\frac{a\,dx}{(a+b\sin x)^{n+1}}+\int\frac{b\sin x\,dx}{(a+b\sin x)^{n+1}}.$$

Let $u=1\div(a+b\sin x)^{n+1}$, $dv=b\sin x\,dx$; then

$$du=-b(n+1)\frac{\cos x}{(a+b\sin x)^{n+2}},\qquad v=-b\cos x.$$

$$\therefore\int\frac{dx}{(a+b\sin x)^n}=\int\frac{a\,dx}{(a+b\sin x)^{n+1}}-\frac{b\cos x}{(a+b\sin x)^{n+1}}$$
$$-b^2(n+1)\int\frac{\cos^2 x\,dx}{(a+b\sin x)^{n+2}}.$$

But $$\int\frac{b^2\cos^2 x\,dx}{(a+b\sin x)^{n+2}}=\int\frac{(b^2-b^2\sin^2 x)\,dx}{(a+b\sin x)^{n+2}}=(b^2-a^2)\int\frac{dx}{(a+b\sin x)^{n+2}}$$
$$+2a\int\frac{dx}{(a+b\sin x)^{n+1}}-\int\frac{dx}{(a+b\sin x)^n};$$

$$\therefore\int\frac{dx}{(a+b\sin x)^n}=\int\frac{a\,dx}{(a+b\sin x)^{n+1}}-\frac{b\cos x}{(a+b\sin x)^{n+1}}+(n+1)(a^2-b^2)$$
$$\times\int\frac{dx}{(a+b\sin x)^{n+2}}-2a(n+1)\int\frac{dx}{(a+b\sin x)^{n+1}}+(n+1)\int\frac{dx}{(a+b\sin x)^n}.$$

Transposing, dividing by $(n+1)(a^2-b^2)$ and writing $n-2$ for n, we have finally:

$$\int\frac{dx}{(a+b\sin x)^n}=\frac{b\cos x}{(n-1)(a^2-b^2)(a+b\sin x)^{n-1}}+\frac{(2n-3)a}{(n-1)(a^2-b^2)}$$
$$\times\frac{dx}{(a+b\sin x)^{n-1}}-\frac{n-2}{(n-1)(a^2-b^2)}\int\frac{dx}{(a+b\sin x)^{n-2}}.\quad(1)$$

In a similar manner we may obtain

$$\int\frac{dx}{(a+b\cos x)^n}=-\frac{b\sin x}{(n-1)(a^2-b^2)(a+b\cos x)^{n-1}}+\frac{(2n-3)a}{(n-1)(a^2-b^2)}$$
$$\times\frac{dx}{(a+b\cos x)^{n-1}}-\frac{n-2}{(n-1)(a^2-b^2)}\int\frac{dx}{(a+b\cos x)^{n-2}}.\quad(2)$$

Integration of $\dfrac{dx}{(a+b\tan x)^n}$.

$$\int\frac{dx}{(a+b\tan x)^n}=\int(a\cos x+b\sin x)dx\frac{\sec x}{(a+b\tan x)^{n+1}}.$$

Let $(a\cos x+b\sin x)dx=dv$, $a\sin x-b\cos x=v$; $u=\dfrac{\sec x}{(a+b\tan x)^{n+1}}$,

$$\therefore du=\left[\frac{\sec x\tan x}{(a+b\tan x)^{n+1}}-b(n+1)\frac{\sec^3 x}{(a+b\tan x)^{n+2}}\right]dx;$$

$$\text{and } vdu = \left[\frac{a\tan^2 x}{(a+b\tan x)^{n+1}} - ab(n+1)\frac{\tan x(1+\tan^2 x)}{(a+b\tan x)^{n+2}} - \frac{b\tan x}{(a+b\tan x)^{n+1}} + b^2(n+1)\frac{(1+\tan^2 x)}{(a+b\tan x)^{n+2}}\right]dx$$

$$= \left[\frac{b^2(n+1)}{(a+b\tan x)^{n+2}} - \frac{b\tan x}{(a+b\tan x)^{n+1}} - \frac{ab(n+1)\tan x}{(a+b\tan x)^{n+2}} + \frac{a\tan^2 x}{(a+b\tan x)^{n+1}} + \frac{b^2(n+1)\tan^2 x}{(a+b\tan x)^{n+2}} - \frac{ab(n+1)\tan^3 x}{(a+b\tan x)^{n+2}}\right]dx.$$

Now

$$\frac{\tan x}{(a+b\tan x)^{n+1}} = \frac{1}{b}\left[\frac{1}{(a+b\tan x)^n} - \frac{a}{(a+b\tan x)^{n+1}}\right],$$

$$\frac{\tan x}{(a+b\tan x)^{n+2}} = \frac{1}{b}\left[\frac{1}{(a+b\tan x)^{n+1}} - \frac{a}{(a+b\tan x)^{n+2}}\right],$$

$$\frac{\tan^2 x}{(a+b\tan x)^{n+1}} = \frac{1}{b^2}\left[\frac{1}{(a+b\tan x)^{n-1}} - \frac{2a}{(a+b\tan x)^n} + \frac{a^2}{(a+b\tan x)^{n+1}}\right]$$

$$\frac{\tan^2 x}{(a+b\tan x)^{n+2}} = \frac{1}{b^2}\left[\frac{1}{(a+b\tan x)^n} - \frac{2a}{(a+b\tan x)^{n+1}} + \frac{a^2}{(a+b\tan x)^{n+2}}\right]$$

$$\frac{\tan^3 x}{(a+b\tan x)^{n+2}} = \frac{1}{b^3}\left[\frac{1}{(a+b\tan x)^{n-1}} - \frac{3a}{(a+b\tan x)^n} + \frac{3a^2}{(a+b\tan x)^{n+1}} - \frac{a^3}{(a+b\tan x)^{n+2}}\right].$$

$$\text{Hence } vdu = \frac{b^2(n+1)dx}{(a+b\tan x)^{n+2}} - \frac{dx}{(a+b\tan x)^n} + \frac{adx}{(a+b\tan x)^{n+1}}$$
$$- \frac{a(n+1)dx}{(a+b\tan x)^{n+1}} + \frac{a^2(n+1)dx}{(a+b\tan x)^{n+2}} + \frac{a}{b^2}\frac{dx}{(a+b\tan x)^{n-1}}$$
$$- \frac{2a^2}{b^2}\frac{dx}{(a+b\tan x)^n} + \frac{a^3}{b^2}\frac{dx}{(a+b\tan x)^{n+1}} + \frac{(n+1)dx}{(a+b\tan x)^n}$$
$$- \frac{2a(n+1)dx}{(a+b\tan x)^{n+1}} + \frac{a^2(n+1)dx}{(a+b\tan x)^{n+2}} - \frac{a(n+1)}{b^2}\frac{dx}{(a+b\tan x)^{n-1}}$$
$$+ \frac{3a^2(n+1)}{b^2}\frac{dx}{(a+b\tan x)^n} - \frac{3a^3(n+1)}{b^2}\frac{dx}{(a+b\tan x)^{n+1}}$$
$$+ \frac{a^4(n+1)}{b^2}\frac{dx}{(a+b\tan x)^{n+2}}.$$

Condensing we find:—

$$\int vdu = \frac{n+1}{b^2}(a^2+b^2)^2\int\frac{dx}{(a+b\tan x)^{n+2}} - \frac{a}{b^2}(3n+2)(a^2+b^2)$$
$$\times\int\frac{dx}{(a+b\tan x)^{n+1}} + \frac{a^2}{b^2}(3n+1)\int\frac{dx}{(a+b\tan x)^n} - \frac{an}{b^2}\int\frac{dx}{(a+b\tan x)^{n-1}}.$$

Hence $\int\frac{dx}{(a+b\tan x)^n}=\int udv=uv-\int vdu$

$$=\int\frac{a\tan x-b}{(a+b\tan x)^{n+1}}-\frac{n+1}{b^2}(a^2+b^2)^2\int\frac{dx}{(a+b\tan x)^{n+2}}+\frac{a}{b^2}(3n+2)$$

$$\times(a^2+b^2)\int\frac{dx}{(a+b\tan x)^{n+1}}-\frac{a^2}{b^2}(3n+1)\int\frac{dx}{(a+b\tan x)^n}+\frac{an}{b^2}\int\frac{dx}{(a+b\tan x)^{n-1}}$$

Transposing, dividing and writing $n-2$ for n:—

$$\int\frac{dx}{(a+b\tan x)^n}=\frac{b^2}{(n-1)(a^2+b^2)^2}\frac{a\tan x-b}{(a+b\tan x)^{n-1}}+\frac{3n-4}{n-1}\frac{a}{a^2+b^2}$$

$$\times\int\frac{dx}{(a+b\tan x)^{n-1}}-\frac{3a^2(n-2)+(a^2+b^2)}{(n-1)(a^2+b^2)^2}\int\frac{dx}{(a+b\tan x)^{n-2}}$$

$$+\frac{n-2}{n-1}\frac{a}{(a^2+b^2)^2}\int\frac{dx}{(a+b\tan x)^{n-3}}.\qquad(3)$$

In a similar manner we may deduce:—

$$\int\frac{dx}{(a+b\cot x)^n}=-\frac{b^2}{(n-1)(a^2+b^2)^2}\frac{a\cot x-b}{(a+b\cot x)^{n-1}}+\frac{3n-4}{n-1}\frac{a}{a^2+b^2}$$

$$\times\int\frac{dx}{(a+b\cot x)^{n-1}}-\frac{3a^2(n-2)+(a^2+b^2)}{(n-1)(a^2+b^2)^2}\int\frac{dx}{(a+b\cot x)^{n-2}}$$

$$+\frac{n-2}{n-1}\frac{a}{(a^2+b^2)^2}\int\frac{dx}{(a+b\cot x)^{n-3}}.\qquad(4)$$

Integration of $\frac{dx}{(a+b\sec x)^n}$.

$$\int\frac{dx}{(a+b\sec x)^n}=\int\frac{adx}{(a+b\sec x)^{n+1}}+b\int\sec x dx\frac{1}{(a+b\sec x)^{n+1}}$$

$$=\int\frac{adx}{(a+b\sec x)^{n\div1}}+b\int\operatorname{cosec}x\cot x\,dx\,\frac{\tan^2x}{(a+b\sec x)^{n+1}}.$$

Let $\operatorname{cosec}x\cot x\,dx=dv$, $-\operatorname{cosec}x=v$;

$$\frac{\tan^2x}{(a+b\sec x)^{n+1}}=u,\ \frac{2\tan x\sec^2x\,dx}{(a+b\sec x)^{n+1}}-\frac{\tan^2x(n+1)b\sec x\tan x\,dx}{(a+b\sec x)^{n+2}}=du;$$

$$\therefore\ vdu=-\frac{2\sec^3x\,dx}{(a+b\sec x)^{n+1}}+(n+1)b\frac{\sec^2x(\sec^2x-1)dx}{(a+b\sec x)^{n+2}}.$$

[To be continued.]

VALUE OF e (NAPIERIAN BASE) TO 137 DECIMALS, COMPUTED BY J. M BOORMAN, ESP.—

e = 2.718281 828459 045235 360287 471352 662497 757247 093699
959574 966967 627724 076630 353547 594571 382178 525166
427427 466391 932003 059921 817413 596629 04357.